Review Fundamentals of Valuation

These class notes review this material and also provide some help for a financial calculator. It also has some self-test questions and problems. Class notes are necessarily brief. See any principles of finance book for a more extensive explanation.

Eugene F. Brigham, Joel F. Houston ***Fundamentals of financial management*** HG 4026 B6693 1998
Ross, Stephen A, Westerfield, and Jordan ***Fundamentals of corporate finance*** HG 4026 .R677 1995

PART I: Single Sum.

Time Value of Money: Know this terminology and notation

FV	Future Value	$(1+i)^t$ Future Value Interest Factor [FVIF]
PV	Present Value	$1/(1+i)^t$ Present Value Interest Factor [PVIF]
i	Rate per period	
t	# of time periods	

Question: Why are (1+i) and $(1+i)^t$ called interest factors?

Answer: 1. Start with simple arithmetic problem on interest:

How much will $10,000 placed in a bank account paying 5% per year be worth compounded annually?

Answer: Principal + Interest
$10,000 + $10,000 x .05 = $10,500

2. Factor out the $10,000.
10,000 x (1.05) = $10,500

3. This leaves (1.05) as the factor.

1. Find the value of $10,000 earning 5% interest per year after **two** years.
   Start with the amount after one year and multiply by the factor for each year.
   [Amount after one year**]** x (1.05)
   = **[**$10,000 x (1.05)**]** x (1.05)
   = $\$10,000 \times (1.05)^2$
   = $11,025.

So $(1+i)^t = (1+i)\cdot(1+i)\cdot(1+i)\cdot(1+i)\cdot(1+i)\cdot(1+i)\cdot(1+i)\ldots\cdot(1+i)$ for "t" times

A. Future Value

Find the value of $10,000 in 10 years. The investment earns 5% per year.

FV = $10,000·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)
FV = $10,000·(1.05)·(1.05)·(1.05)·(1.05)·(1.05)·(1.05)·(1.05)·(1.05)·(1.05)·(1.05)
FV = $10,000 x $(1.05)^{10}$
= $10,000 x 1.62889
= $16,289

Find the value of $10,000 in 10 years. The investment earns 8% for four years and then earns 4% for the remaining six years.
FV = $10,000·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)
FV = $10,000·(1.08)·(1.08)·(1.08)·(1.08)·(1.04)·(1.04)·(1.04)·(1.04)·(1.04)·(1.04)
FV = $10,000 x $(1.08)^4$ x $(1.04)^6$
FV = $17,214.53

B. Present Value:

Same idea, but begin at the end. Rearrange the Future value equation to look like this:

PV = FV÷ [(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)·(1+i)]
PV = FV ÷ $(1+i)^t$ [2]

Example: How much do I need to invest at 8% per year, in order to have $10,000 in__.

a. One year:	**PV =10,000 ÷ (1.08) = $9,259.26**	
b. Two years:	**PV = $10,000 ÷ (1.08) ÷ (1.08)**	
	OR $10,000 ÷ $(1.08)^2$ = $8,573	
c. Ten years	**PV = $10,000 ÷ $(1.08)^{10}$ = $10,000 ÷ 2.1589 = $4,632**	

C. Rate of Return

START WITH SAME RELATIONHSIP: FV = PV x $(1+i)^t$

Solve for i.

$(1+i)^t$ =FV/PV.

1+i = $(FV/PV)^{1/t}$

i = $(FV/PV)^{1/t}$-1.

Question: An investor deposits $10,000. Ten years later it is worth $17,910. What rate of return did the investor earn on the investment?

Solution:

$17,910 = $10,000 x $(1+i)^{10}$
$(1+i)^{10}$ = $17,910/10,000 = 1.7910
(1+i) = $(1.7910)^{1/10}$ = 1.060
i = .060 = 6.0%

D. Finding the Future Value

Find the value of $10,000 today at the end of 10 periods at 5% per period.

KEY RELATIONHSIP:

$$FV = PV \times (1+i)^t$$

1. Scientific Calculator:

Use [y^x] y = (1+i) = 1.05 and x =t= 10.

1. Enter 1.05.
2. Press **[y^x].**
3. Enter the exponent.
4. Enter [=].
5. Multiply result by $10,000.

2. Spreadsheet:

	A	B
1		
2		
3	Interest Rate	5%
4	Present Value	10,000
5	Periods	10
6	Future Value	$ 16,288.95
7		
8	Formula in Cell B6	=B4*(1+B3)^B5
9	Alternative	=-FV(B3,B5,0,B4)
10		

3. Financial calculator. You may need to input something like this.

Specific functions vary. Be sure to consult the calculator's manual!!!!!!

	n [N]	i [I/YR]	PV	PMT	FV
	10	5	10,000	0	?

NOTE: The future value will be negative, indicate an opposite direction of cash flow.

1. Set the calculator frequency to once per period.
2. Enter negative numbers using the [+/-] key, not the subtraction key.
3. Be sure the calculator is set in the END mode.

E. Fundamental Idea.

Question: What is the value of any financial asset?

Answer: The present value of its expected cash flows.

F. Finding the Present Value

Find the present value of $10,000 to be received at the end of 10 periods at 8% per period.

a. Scientific Calculator

KEY RELATIONHSIP:

$$PV = FV \div (1+i)^t$$

Scientific Calculator:

Use [y^x] where y = 1.08 and x = -1,-2, or -10.

1. Enter 1.08.
2. Press **[y^x]**
3. Enter the exponent as a negative number
4. Enter [=].
5. Multiply result by $10,000.

b. Spreadsheet

	A	B
1		
2		
3	Interest Rate	8%
4	Present Value	10,000
5	Periods	10
6	Present Value	$ 4,631.93
7		
8	Formula in Cell B6	=B4*(1+B3)^-B5
9	Alternative	=-PV(B3,B5,0,B4)
10		

c. Financial calculator. You may need to input something like this.

Specific functions vary. Be sure to consult the calculators' manual!!!!!!

	n [N]	i [I/YR]	PV	PMT	FV
c.	10	8	10,000	0	?

The present value will be negative, to indicate the opposite direction of cash flow.

G. Finding the [geometric average] rate of return:

KEY RELATIONHSIP:

$$(1+i)^t = FV \div PV$$
$$(1+i) = (FV \div PV)^{1/t}$$

Scientific Calculator

To find i, use [y^x] and [1/x].

1. Enter **1.7910**,
2. Press **[y^x]**
3. Enter the exponent **10** then press **[1/x]**
4. Press **[=]**.
5. Subtract 1

2. Spreadsheet

	A	B
1		
2	Future Value	17,910
3	Present Value	10,000
4	Periods	10
5		6.0%
6	Formula in Cell B5 =	=(B2/B3)^(1/B4)-1

3. Financial Calculator. (Your financial calculator may differ. Consult your manual.)

n [N]	i [I/YR]	PV	PMT	FV
10	?	-10,000	0	17,910

Answer i = 6%

- **Question: Today your stock is worth $50,000. You invested $5,000 in the stock 18 years ago. What average annual rate of return [i] did you earn on your investment?**
  Answer: 13.646%.

- **Question: The total percentage return was 45,000÷5000=900%. Why doesn't the average rate of return equal 50%, since 900%÷18 = 50%?**

H. FUTURE VALUE WHEN RATES OF INTEREST CHANGE.

$FV = PV \times (1+i_1) \times (1+i_2) \times (1+i_3) \times \ldots \times (1+i_t)$.

Example:

You invest $10,000. During the first year the investment earned 20% for the year. During the second year, you earned only 4% for that year. How much is your original deposit worth at the end of the two years?

$FV = PV \times (1+i_1) \times (1+i_2)$
= $10,000 x (1.20`) x (1.04) = $12,480.

Question:
The arithmetic average rate of return is 12%, what is the geometric average rate of return?

Answer:

An average rate of return is a geometric average since it is a rate of growth. The 12% is the arithmetic average. The geometric average rate of return on the investment was 11.7%.

$$i = (FV/PV)^{1/t}-1 = (12{,}480/10000)^{1/2}-1 = .1171$$

OR $$i = \sqrt{(1.20)\cdot(1.04)} - 1 = 0.1171$$

Important: Although 20% and 4% average to 12%, the $10,000 not grow by 12%. [$10,000 x $(1.12)^2$= 12,544 **NOT** $12,480].

I. COMPOUNDING PERIODS

Up to this point, we have used years as the only time period. Actually, all the previous examples could have been quarters, months, or days.
The interest rate and time period must correspond.

Example:

Problem 1.
Find the value of $10,000 earning 5% interest per year after two years.

Problem 2.
Find the value of $10,000 earning 5% interest per quarter after two quarters.

Both problems have same answer

$10,000 x $(1.05)^2$ = $11,025.

However:

In the first problem t refers to years and i refers to interest rate per year.
In the second problem t refer to quarters and i to interest rate per quarter.

$FV_t = PV \times (1+i)^t$.
t = number of periods
i = interest for the period.

Alternatively,

$FV_{t\bullet m}$ = PV x $(1+i/m)^{t\bullet m}$.

m= periods per year,
t= number of years,
i = the interest per year [APR].

Example:
What will $1,000 be worth at the end of one year when the annual interest rate is 12% [This is the **APR**.] when interest is compounded:

Annually: **t=1** **i =12%** FV_1 = PV x $(1+i)^1$ = $1,000 x $(1.12)^1$ = $1,120.
Quarterly: t=4 **i = 3%** FV_4 = PV x $(1+i)^4$ = $1,000 x $(1.03)^4$ = $1,125.51.
Monthly: t=12 **i =1%** FV_{12} = $1,000 x $(1.01)^{12}$ = $1,000 x (1.126825) = $1,126.825.

Daily: t=365 i = (12%÷365) = 0.032877%
FV_{365} = $1,000 x $(1.00032877)^{365}$= $1,000 x (1.12747) = $1,127.47.

n [N]	i [I/YR]	PV	PMT	FV
1	12	1,000	0	?
4	3	1,000	0	?
12	1	1,000	0	?
365	.032877	1,000	0	?

How about compounding at every instant?

E. CONTINUOUS COMPOUNDING: [Used in Black Scholes option pricing model.]

$$\lim_{m \to \infty} \left[1 + \frac{i}{m}\right]^{t \cdot m} = e^{i t}$$

Example: What is $1,000 worth in one year if compounded at 12% continuously.

FV = $1,000 x $e^{.12}$
= $1,000 x 1.127497 = $1,127.50

This is $.03 more than daily compounding.

Try this on your calculator. Find the e^x button. $e^{.12}$ = 1.12749
Present Value Interest Factor = [$e^{-i\ t}$]
Problem: What is the present value of $10,000 to be received 3 years from today compounded continuously at 10%?PV = $10,000 x $e^{-.10 \times 3}$ = $10,000 x 0.74082=$7,408
Try this on your calculator. Find the e^x button. $e^{-0.3}$ = 0.74082

Practice Quiz Questions: PV and FV of a Single sum.

<u>Review Problems</u>

1. How much must you deposit today in a bank account paying interest compounded quarterly:

   a. if you wish to have $10,000 at the end of 3 months, if the bank pays 5.0% APR?
   Answer: $9,877

   b. if you wish to have $50,000 at the end of 24months, if the bank pays 8.0%APR?
   Answer: $42,675

   c. if you wish to have $6,000 at the end of 12 months, if the bank pays 9.0% APR?
   Answer: $5,489

2. a. What rate of interest [APR] is the bank charging you if you borrow $77,650 and must repay $80,000 at the end of 2 quarters, if interest is compounded quarterly?
   Answer: 6.0% APR

   b. What rate of interest [APR] is the bank charging you if you borrow $49,000 and must repay $50,000 at the end of 3 months, if interest is compounded monthly?
   Answer: 8.0% APR

3. How much must you deposit today in a bank account paying interest compounded monthly:

   a. if you wish to have: $10,000 at the end of 1 months, if the bank pays 5.0% APR ?
   Answer: $9,959

   b. if you wish to have: £6,000 at the end of 6 months, if the bank pays 9.0% APR ?
   Answer: £5,737

   c. if you wish to have: $12,000 at the end of 12 months, if the bank pays 6.0% APR ?
   Answer: $11,303

4. If interest is compounded <u>quarterly</u>, how much will you have in a bank account:

   a. if you deposit today £8,000 at the end of 3 months, if the bank pays 5.0% APR ?
   Answer: £8,100

   b. if you deposit today $10,000 at the end of 6 months, if the bank pays 9.0% APR ?
   Answer: $10,455

   c. if you deposit today ¥80,000 at the end of 12 months, if the bank pays 8.0% APR ?
   Answer: ¥86,595

   d. if you deposit today $5,000 at the end of 24 months, if the bank pays 5.0% APR ?
   Answer: $5,522

5. If interest is compounded <u>monthly</u>, how much will you have in a bank account,

   a. if you deposit today £8,000 at the end of 3 months, if the bank pays 5.0% APR ?
   Answer: £8,100

   b. if you deposit today $10,000 at the end of 6 months, if the bank pays 9.0% APR ?
   Answer: $10,459

   c. if you deposit today ¥80,000 at the end of 12 months, if the bank pays 8.0% APR ?
   Answer: ¥86,640

   d. if you deposit today £5,000 at the end of 24 months, if the bank pays 5.0% APR ?
   Answer: £5,525

6. You borrowed $1,584 and must repay $2,000 in exactly 4 years from today. Interest is compounded annually.
   a. What is the interest rate [APR] of the loan? Answer 6.0%
   b. What effective annual rate [EAR] are you paying? Answer 6.0%

7. You now have $8,000 in a bank account in which you made one single deposit $8,000 monthly of $148.97 exactly 40 years ago. Interest is compounded monthly.
   a. What rate of interest [APR] is the bank paying? Answer 10.0%
   b. What effective annual rate [EAR] is the bank paying? Answer 10.47%

<u>Possibly New Problems.</u>

8. Suppose you make an investment of $1,000. This first year the investment returns 12%, the second year it returns 6%, and the third year in returns 8%. How much would this investment be worth, assuming no withdrawals are made?

Answer:
1000*(1.12) x (1.06) x (1.08)
= $1,282

9. Why is (1+i) called an interest factor?

Factoring the expression $10,000 + 10,000 x i = 10,000 x (1+i)
Thus (1+i) is an interest factor.

10. Suppose you make an investment of $1,000. This first year the investment returns 5%, the second year it returns i. Write an expression, using i, that represents the future value of the investment at the end of two years.

Answer:
FV=1,000 x (1.05) x (1+i)

11. An investment is worth $50,000 today. This first year the investment returns 9%, the second year it returns i. Write an expression using i that represents the original value of the investment.

Answer:
PV=50,000÷[(1.09) x (1+i)]

12. Suppose you make an investment of $A. This first year the investment returns 10%, the second year it returns 16%, and the third year in returns 2%. How much would this investment be worth, assuming no withdrawals are made?

Answer:
A*(1.10) x (1.16) x (1.02)

11. Suppose you make an investment of $10,000. This first year the investment returns 15%, the second year it returns 2%, and the third year in returns 10%. How much would this investment be worth at the end of three years, assuming no withdrawals are made?

$12,903

12. Refer to the above problem. What is the <u>geometric</u> average rate of return?

8.9%

Review Fundamentals of Valuation

Part II Multiple Periods: Uneven and Even (Annuities)

- Periodic Uneven Cash Flows

What is the value of the following set of cash flows today? The interest rate is 8% for all cash flows.

Year and Cash Flow
1: $ 300 2: $ 500 3: $ 700 4: $ 1000

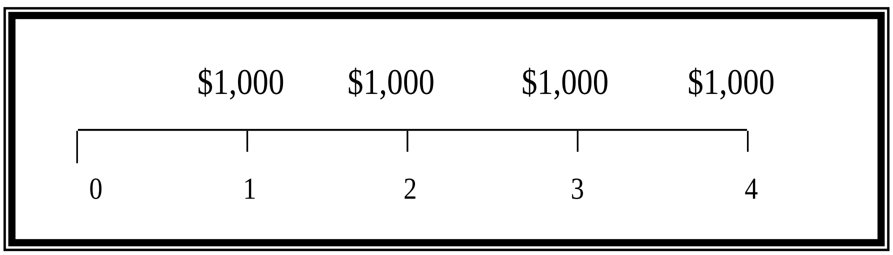
- Solution: Find Each Present Value and Add

$$\frac{300}{1.08^1} + \frac{500}{1.08^2} + \frac{700}{1.08^3} + \frac{1000}{1.08^4} =$$

277.78 428.67 555.68 735.03 = 1997.16

- Periodic Cash Flow: Even Payments

An annuity is a level series of payments. For example, four annual payments, with the first payment occurring exactly one period in the future is an example of an ordinary annuity.



A. Present value of an annuity:

The present value of each of the cash flows is the value of the annuity. This could be done one at a time, but this might be tedious.

Annuity Present Value Interest Factor

$$PVIFA = [1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^t]$$

$$PVIFA = \sum_{j=1}^{t} 1/(1+i)^j$$

$$PVIFA = \frac{1 - 1/(1+i)^t}{i}.$$

Example:
What is the present value of a 4-year annuity, if the annual interest is 5%, and the annual payment is $1,000?

i = 5%; PMT = $1,000; t =4; PV = ?

$PV = 1{,}000/(1.05) + 1{,}000/(1.05)^2 + 1{,}000/(1.05)^3 + 1{,}000/(1.05)^4$ ← **Long way.**

Factor out the single sum interest rate factors:
$PV = 1{,}000 \times [1/(1.05) + 1/(1.05)^2 + 1/(1.05)^3 + 1/(1.05)^4] =$

$PV = 1{,}000 \times [PVIFA\ (_{4,5\%})] =$ ← **Short Way**

Calculate: $PVIFA(_{4,5\%}) = \dfrac{1-1/(1+i)^t}{i} = \dfrac{1-PVIF_{4,5\%}}{5\%}\ \ \dfrac{1-0.8227}{.05} = 3.54595.$

$PV = 1{,}000 \times [3.5460] = \$3{,}546.$

Finding the Future Value of an annuity on a:

1. <u>Scientific Calculator.</u>
   To calculate PVIFA using scientific calculator:

FIRST FIND: $PVIF_{4,5\%} = 1/(1+i)^t = 1/(1.05)^4 = 0.82270$

THEN FIND: $PVIFA(_{4,5\%}) = \dfrac{1-1/(1+i)^t}{i} = \dfrac{1-PVIF}{i}\ \ \dfrac{1-0.8227}{.05} = 3.54595.$

$= 1{,}000 \times [3.5460] = \$3{,}546.$

2. **Using a spreadsheet.**

	A	B
1		
2	**Rate**	5%
3	**Payment**	-1000
4	**Periods**	4
5	**Future Value**	0
6	**Present Value**	$3,545.95
7	Formula in B 6	=PV(B2,B4,B3,B5)

3. **Using a financial calculator, the Present Value of an annuity.**

n [N]	i [I/YR]	PV	PMT	FV
4	5	?	-1000	0

PV= $3,546.

Note: Most financial calculators require i [I/YR] to be a percentage. That is enter a 5, not .05. However, Excel requires .05 or 5%.

B. Future value of an annuity:

> Note: Last Payment earns no interest in ordinary annuity. The interest factor on that payment is 1. The first payment earns interest for t-1 periods, not t periods.

- **Annuity Future Value Interest Factor**
  $FVIFA = [1 + (1+i) + (1+i)^2 + ... + (1+i)^{t-1}]$.

$$FVIFA = \sum_{j=0}^{t-1} (1+i)^t$$

$$FVIFA = \frac{(1+i)^t - 1}{i}$$

Example: What is the future value of a 4-year annuity, if the annual interest is 5%, and the annual payment is $1,000?
i = 5%; PMT = $1,000; t =4; FV = ?

$\$1,000 \times [1 + (1.05) + (1.05)^2 + (1.05)^3] =$
$\$1,000 \times [FVIFA(4,5\%)] =$
$\$1,000 \times [4.3101] = \$4,310.1$

Finding FVIFA
1. Using scientific calculator:

FIRST FIND: $FVIF = (1+i)^t = (1.05)^4 = 1.2155$

THEN: $FVIFA(t,i) = \frac{(1+i)^t - 1}{i} = \frac{FVIF - 1}{i}$

> **Use Short Cut Formula**

$FVIFA(t,i) = \frac{FVIF_{4,5\%} - 1}{5\%} = \frac{1.2155-1}{.05} = 4.3101$

2. Using a Spreadsheet

	A	B
1		
2	**Rate**	5%
3	**Payment**	-1000
4	**Periods**	4
5	**Present Value**	0
6	**Future Value**	$4,310.13
7	Formula in B 6	=FV(B2,B4,B3,B5)

3. Using a financial calculator, the Future Value of an annuity:

n [N]	i [I/YR]	PV	PMT	FV
4	5	0	-1000	?

FV = $4,310

Question: How much would you need to deposit every month in an account paying 6% a year to accumulate by $1,000,000 by age 65 beginning at age 20?

Data: FV = $1,000,000 PMT = ?
i = 6%÷12 = 0.5% per month
n = (65-20) x 12 = 45 x 12 = 540 months.

Answer: PMT = $362.85

C. RATE OF RETURN OF AN ANNUITY

- You borrow $60,000 and repay in 8 equal annual installments of $12,935 with the first payment made exactly 1 year later. To the nearest percent, what rate of interest are you paying on your loan? **Difficult without financial calculator. Can use table to find answer to the nearest percent.**

Data:
i = ? PV = $60,000 PMT=$12,935 t = 8 years

Relationship: PV = PMT x PVIFA(t, i)

1. Solution: (Trial and Error with Table)
PVIFA(t, i) = PV/PMT= $60,000/12,935 = 4.6386

Table
..... 14% ....
:
8 .. 4.6389 .. :

Therefore: PVIFA = 4.6386. S_o> i = 14%

2. (Trial and Error using a spreadsheet program)

	A	B
1		
2	**Periods**	8
3	**Payment**	12,935
4	**Present Value**	-60,000
5	**Future Value**	0
6	**Type**	0
7	**Guess**	0.1
8	**Rate**	**14.00%**
9	Formula in Cell B8	=RATE(B2,B3,B4,B5,B6,B7)

3. (Trial and Error using a financial calculator)

n [N]	i [I/YR]	PV	PMT	FV
8	?	60000	-12935	0

i = 14%

D. Example of Annuity with quarterly compounding:

An investment of $3000 per quarter for 6 years at annual interest rate of 8%, compounded quarterly, will accumulate by the end of year 6 to:

Solution:

FV = ? PMT = $3,000 t = 24 i = 2%

FV = PMT x FVIFA (t, i).

FV = $3,000 x [30.422] = $91,266.

n [N]	i [I/YR]	PV	PMT	FV
24	2	0	-3000	?

$91,266

Review Problems with solutions.

1. This one: is a typical mortgage problem. You borrow $80,000 to be repaid in equal monthly installments for 30 years. The APR is 9%. What is the monthly payment?

PV = $80,000 i =0.75%,
t=360 PMT = ?

$80,000 = PMT x 124.282
PMT = $643.70

n [N]	i [I/YR]	PV	PMT	FV
360	.75	-80000	?	0

2. Try this one. You make equal $400 monthly payments on a loan. The interest rate equals 15% APR, compounded monthly. The loan is for 12 years. What is the amount of the loan? Answer: PV = $26,651

3. Retire with a million: How much would must you deposit monthly in an account paying 6% a year [APR], compounded monthly, to accumulate $1,000,000 by age 65 beginning at age 30? Answer: PMT = $701.90

n [N]	i [I/YR]	PV	PMT	FV
420	0.50	0	?	1000000

4. Using a financial calculator for annuity calculations:
Calculate the future value of $60.00 per year at 7% per year for eight years.

$$FV = 60\sum_{j=0}^{7}(1+.07)^{j}$$

$$FV = 60 \cdot \frac{(1.07)^{8}-1}{.07}$$

$$FV = \$615.5$$

n [N]	i [I/YR]	PV	PMT	FV
8	7	0	-60	?

FV = $615.50

5. Calculate the future value of $50.00 per month at 6% APR for 24 months

n[N]	i [I/YR]	PV	PMT	FV
24	0.5	0	-50	?

FV = $1,217.60

6. Calculate the present value of $500 per year at 6% per year for 5 years (monthly compounding).

n[N]	i [I/YR]	PV	PMT	FV
5	6	?	-500	0

PV=$2,106

7. You borrow $5,000 and repay the loan with 12 equal monthly payments of $500? Calculate the interest rate per month and the APR.

n[N]	i [I/YR]	PV	PMT	FV
12	?	5,000	-500	0

i = 2.92% per month.
APR = i x 12
APR = 2.92% x 12 = 35.04

8. Problem on inflation.

υ You will receive $100,000 dollars when you retire, forty years from today. If inflation averages 3% per year for the next forty years, how much would that amount be worth measured in today's dollars? (Note, this is **not** a time value of money problem, but it solved with a similar calculation. Such adjustments are necessary to overcome “money illusion”]

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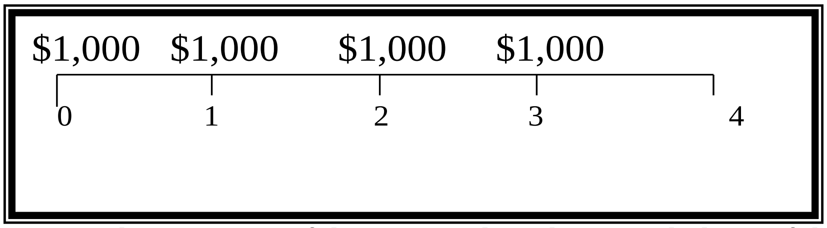
- **Solution:**
  $\$100,000 \div (1.03)^{40} = 100,000 \div 3.26204 = \$\ 30,655$

D. Annuity Due



Question: Compare the payments of the annuity due, above, with those of the ordinary annuity earlier. What is the difference? How does this difference affect its value?

Answer: Each payment in an annuity due occurs one period earlier than it would in ordinary annuity. Both present value and future value of each payment in an annuity due if (1+i) times greater than it would be for an ordinary annuity.

Question: What is the present value of the above four-year annuity due?

$$\$1,000 \times [1 + 1/(1+i) + 1/(1+i)^2 + 1/(1+i)^3]$$
$$= \$1,000 \times (1+i) \times [1/(1+i) + 1/(1+i)^2 + 1/(1+i)^3 + 1/(1+i)^4]$$
$$= \$1,000 \times (1+i) \times PVIFA_{i,4}$$

PV interest factor of an annuity due is: (1+i)·PVIFA
FV interest factor of an annuity due is: (1+i)·FVIFA

Problem. **What is the present value of an annuity due of five \$800 annual payments discounted at 10%?** $800 \times (1.10) \times PVIVA_{10\%,5} =$
$800 \times (1.10) \times 3.79079 \times =$
$800 \times 4.16987 = \$3,335.9$

Note: Financial calculators have a *BEGIN* and *END* mode. The above assumes the *END* mode. If the calculator is set in the *BEGIN* mode, it calculates an annuity due.

Problem. **What is the present value of an annuity of five annual \$800 payments discounted at 10%? The first payment is due in <u>one-half year</u> from today.**

$800 \times (1.10)^{1/2} \times PVIVA_{10\%,5} =$
$800 \times (1.04881) \times 3.79079 \times =$
$800 \times 3.97581 = 3,180.7$